4. A company manufactures model rockets that require igniters to launch. Once an igniter is used to launch a rocket, the igniter cannot be reused. Sometimes an igniter fails to operate correctly, and the rocket does not launch. The company estimates that the overall failure rate, defined as the percent of all igniters that fail to operate correctly, is 15 percent.

A company engineer develops a new igniter, called the super igniter, with the intent of lowering the failure rate. To test the performance of the super igniters, the engineer uses the following process.

      Step 1: One super igniter is selected at random and used in a rocket.

      Step 2: If the rocket launches, another super igniter is selected at random and used in a rocket.

Step 2 is repeated until the process stops. The process stops when a super igniter fails to operate correctly or 32 super igniters have successfully launched rockets, whichever comes first. Assume that super igniter failures are independent.

(a) If the failure rate of the super igniters is 15 percent, what is the probability that the first 30 super igniters selected using the testing process successfully launch rockets?

(b) Given that the first 30 super igniters successfully launch rockets, what is the probability that the first failure occurs on the thirty-first or the thirty-second super igniter tested if the failure rate of the super igniters is 15 percent?

(c) Given that the first 30 super igniters successfully launch rockets, is it reasonable to believe that the failure rate of the super igniters is less than 15 percent? Explain.
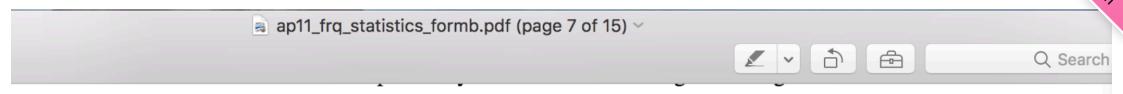
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2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

(a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

(c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

> Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

3. An airline claims that there is a 0.10 probability that a coach-class ticket holder who flies frequently will be upgraded to first class on any flight. This outcome is independent from flight to flight. Sam is a frequent flier who always purchases coach-class tickets.

(a) What is the probability that Sam's first upgrade will occur after the third flight?

(b) What is the probability that Sam will be upgraded exactly 2 times in his next 20 flights?

(c) Sam will take 104 flights next year. Would you be surprised if Sam receives more than 20 upgrades to first class during the year? Justify your answer.

3. A test consisting of 25 multiple-choice questions with 5 answer choices for each question is administered. For each question, there is only 1 correct answer.

(a) Let X be the number of correct answers if a student guesses randomly from the 5 choices for each of the 25 questions. What is the probability distribution of X ?

This test, like many multiple-choice tests, is scored using a penalty for guessing. The test score is determined by awarding 1 point for each question answered correctly, deducting 0.25 point for each question answered incorrectly, and ignoring any question that is omitted. That is, the test score is calculated using the following formula.

Score = (1 × number of correct answers) – (0.25 × number of incorrect answers) + (0 × number of omits)

For example, the score for a student who answers 17 questions correctly, answers 3 questions incorrectly, and omits 5 questions is

$$\text{Score} = (1 \times 17) - (0.25 \times 3) + (0 \times 5) = 16.25.$$

(b) Suppose a student knows the correct answers for 18 questions, answers those 18 questions correctly, and chooses randomly from the 5 choices for each of the other 7 questions. Show that the expected value of the student's score is 18 when using the scoring formula above.

(c) A score of at least 20 is needed to pass the test. Suppose a student knows the correct answers for 18 questions, answers those 18 questions correctly, and chooses randomly from the 5 choices for each of the other 7 questions. What is the probability that the student will pass the test?

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3. At an archaeological site that was an ancient swamp, the bones from 20 brontosaur skeletons have been unearthed. The bones do not show any sign of disease or malformation. It is thought that these animals wandered into a deep area of the swamp and became trapped in the swamp bottom. The 20 left femur bones (thigh bones) were located and 4 of these left femurs are to be randomly selected without replacement for DNA testing to determine gender.

(a) Let X be the number out of the 4 selected left femurs that are from males. Based on how these bones were sampled, explain why the probability distribution of X is <u>not</u> binomial.

(b) Suppose that the group of 20 brontosaurs whose remains were found in the swamp had been made up of 10 males and 10 females. What is the probability that all 4 in the sample to be tested are male?

(c) The DNA testing revealed that all 4 femurs tested were from males. Based on this result and your answer from part (b), do you think that males and females were equally represented in the group of 20 brontosaurs stuck in the swamp? Explain.

(d) Is it reasonable to generalize your conclusion in part (c) pertaining to the group of 20 brontosaurs to the population of all brontosaurs? Explain why or why not.